Michaelis-Menten Model Analysis

ODE (Ordinary Differential Equation):

$$\frac{dx}{dt} = \frac{Vx}{x + k_d} - \frac{bxy}{x + m}$$

$$\frac{dy}{dt} = cxy - dy$$

x	The population of prey at time t
y	The population of predator at time t
V	Maximum Speed of prey production
k_d	Equilibrium Constant
b	Similar to V
m	Similar to k_d
c	Rate constant on how fast predator grow
d	Rate constant on how fast predator die

- The prey (AHL) production is limited by the limited number of promoters. This is modeled by Michaelis-Menten kinetics. The rate of production of the prey and predator becomes linear when number of promoters is saturated.
- The rest parts of the equation are still behaving the same as the original Lotka-Volterra equation.

Stationary Points

- By setting ODEs to 0, i.e. dx/dt=0 & dy/dt=0, there are two stationary points found.

[0, 0]
[d/c, V*(m*c+d)/b/(k*c+d)]

Jacobian Analysis

- The Jacobian matrix of ODEs is

[-b*y/(m+x)+V/(k+x)+x*(b*y/(m+x)^2-V/(k+x)^2), -x*b/(m+x)]
[y*c, -d+c*x]

Stability analysis (eigenvalue)

- For the first stationary point [0, 0], the Jacobian matrix is
  [V/k, 0]
  [0, -d]
- Trace = V/k – d, Determinant = -Vd/k < 0
  → The first stationary point is **unstable**.
- For the second stationary point [d/c, V*(m*c+d)/b/(k*c+d)], the Jacobian matrix is
  [d*V/(m*c+d)*c^2*(k-m)/(k*c+d)^2, -d*b/(m*c+d)]
  [V*(m*c+d)/b/(k*c+d)*c, 0]

Trace = d*V/(m*c+d)*c^2***(k-m)**/(k*c+d)^2

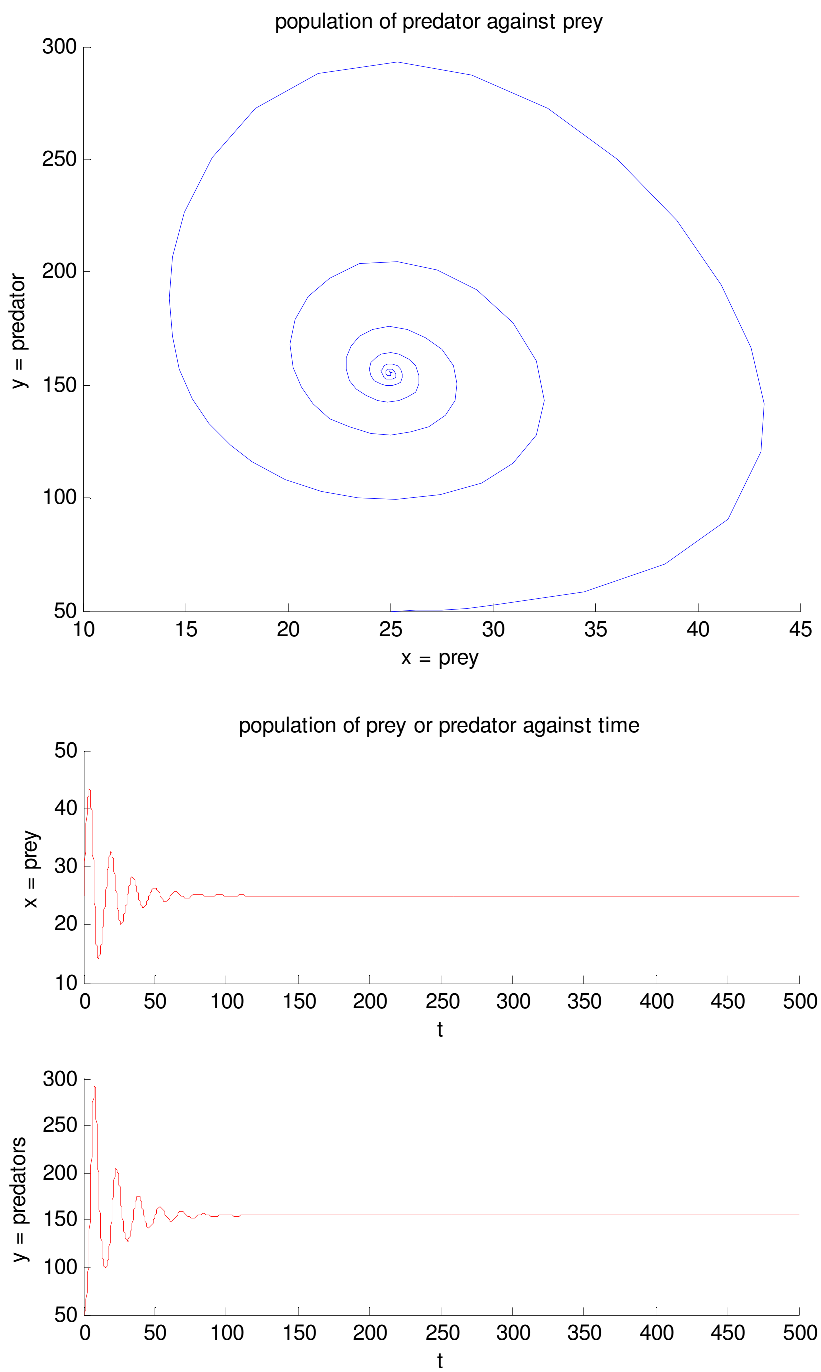
Determinant = d*V/(k*c+d)*c > 0

→ The second stationary point is **stable** if k<m

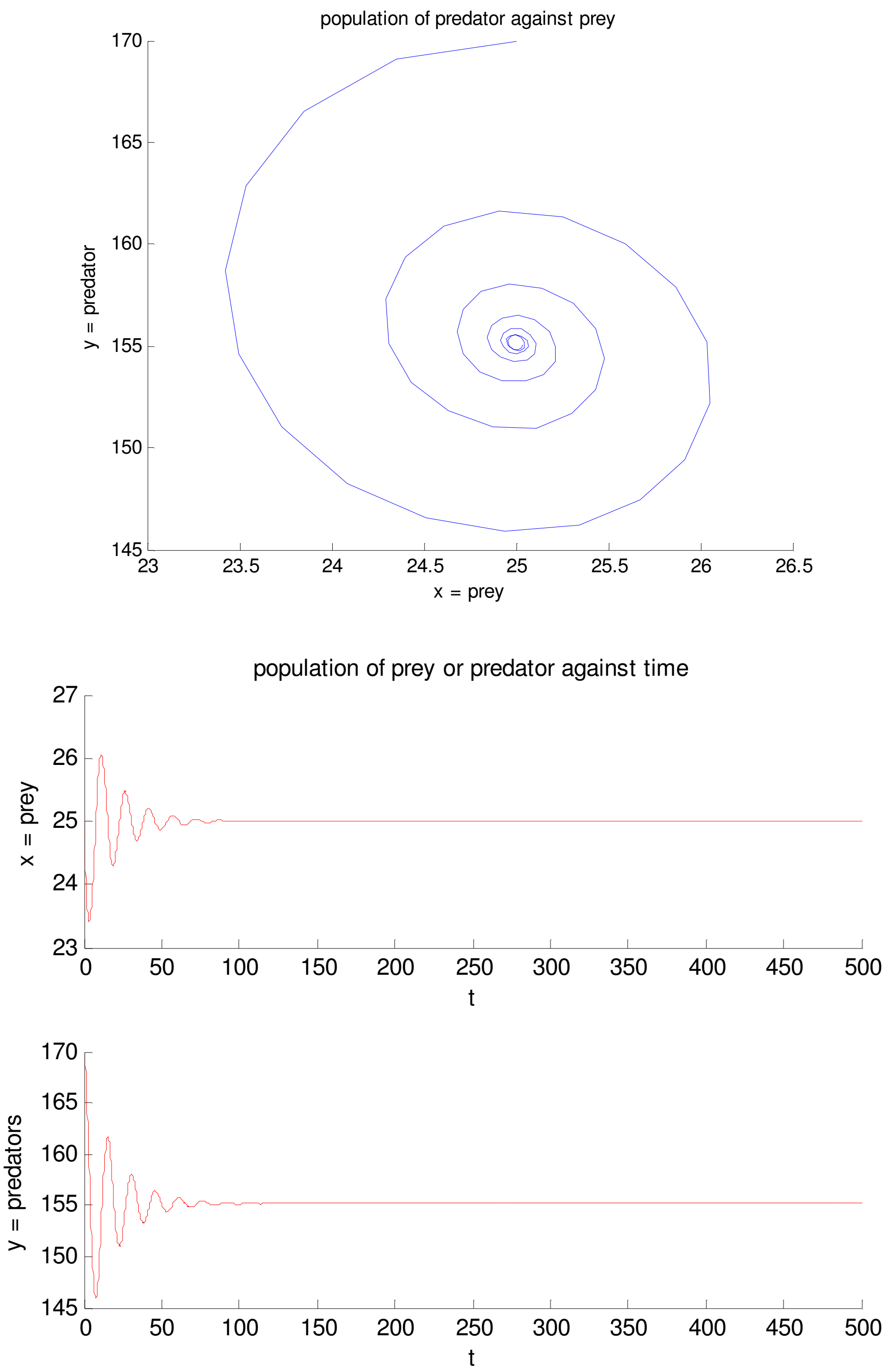
unstable if k>m

Graphic Representation

- The contour of the Predator against Prey is plotted by Matlab. The following graphs are generated
- a = 1 b = 0.1 c = 0.02 d = 0.5 V = 10 k = 4 m = 20 (k<m, stable)

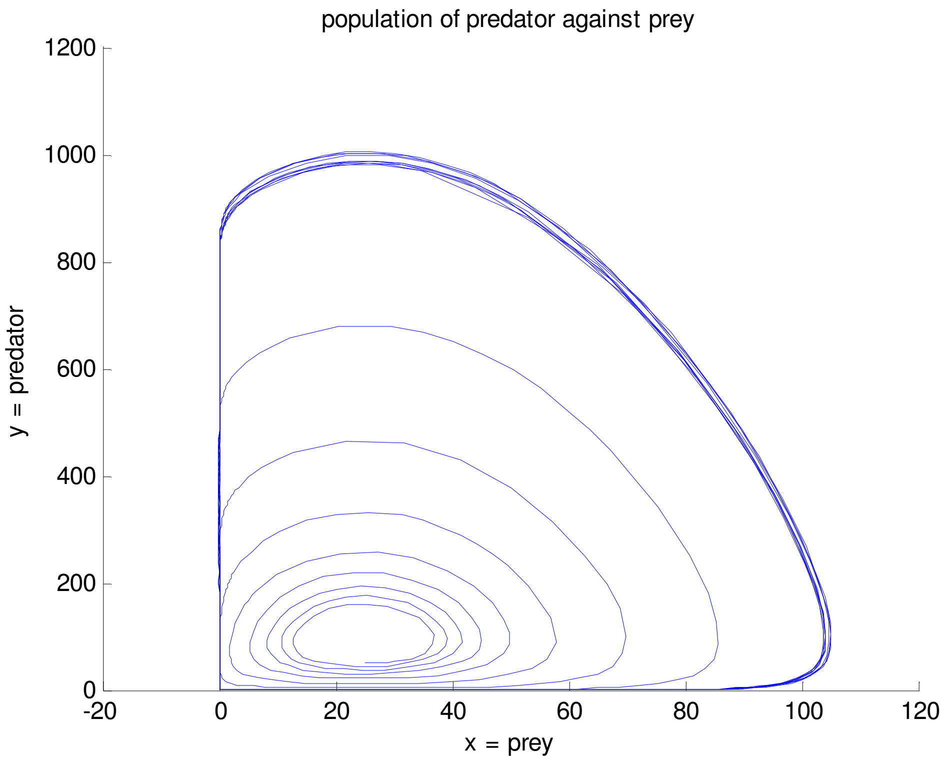


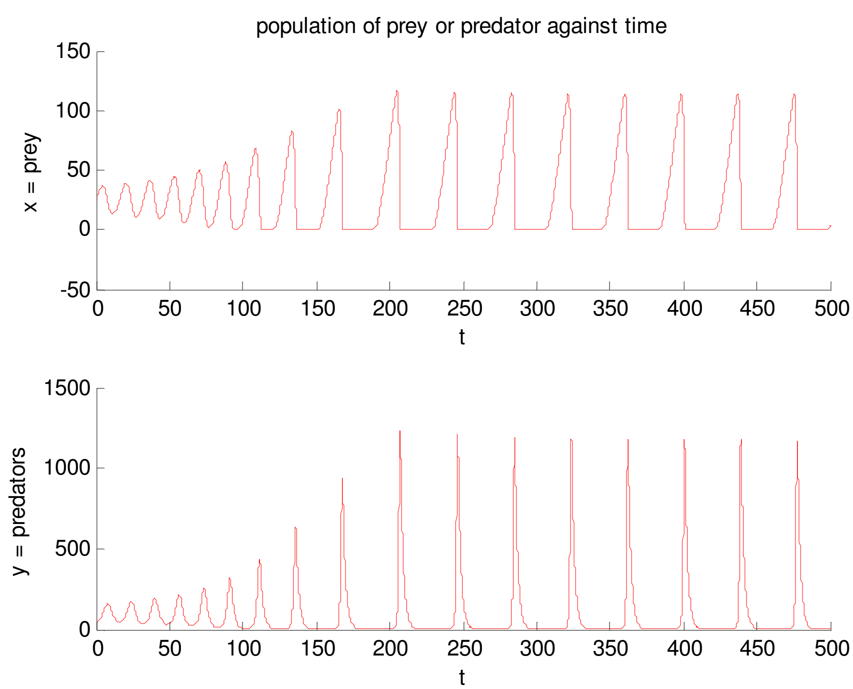
initial condition [25,50]



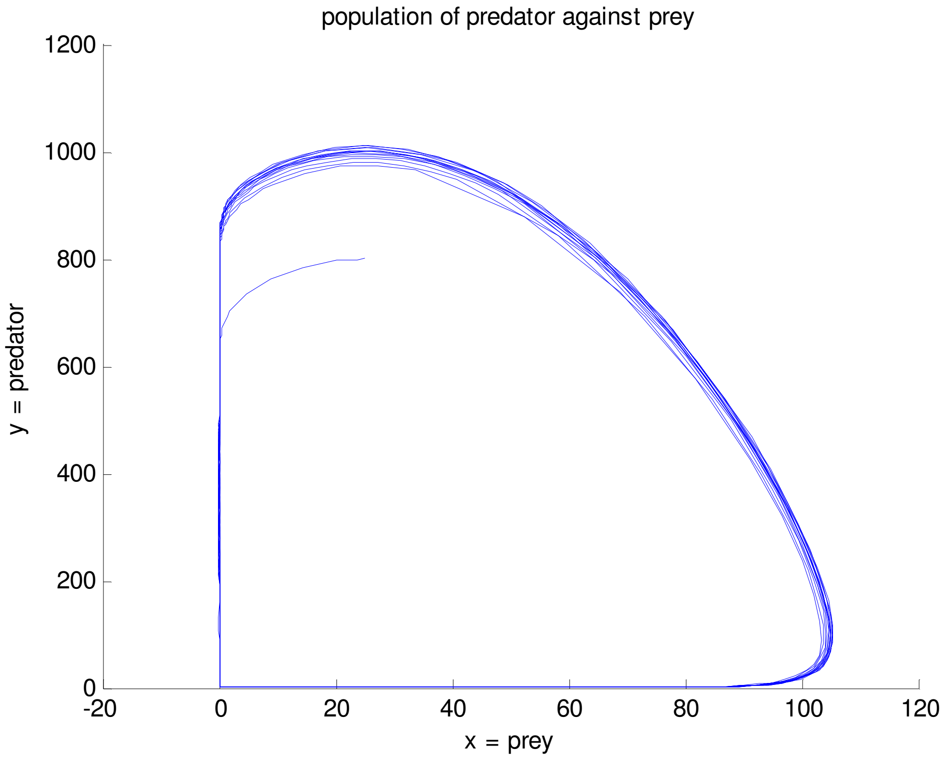
initial condition [25,170]

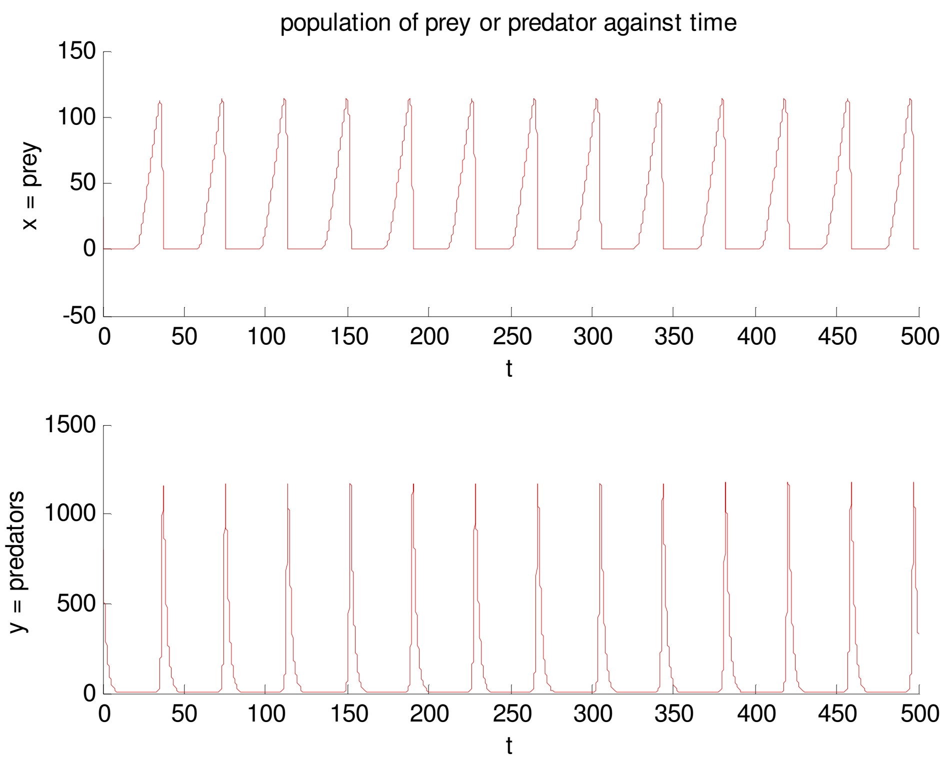
- a = 1 b = 0.1 c = 0.02 d = 0.5 V = 10 k = 6 m = 4 (k>m, unstable)





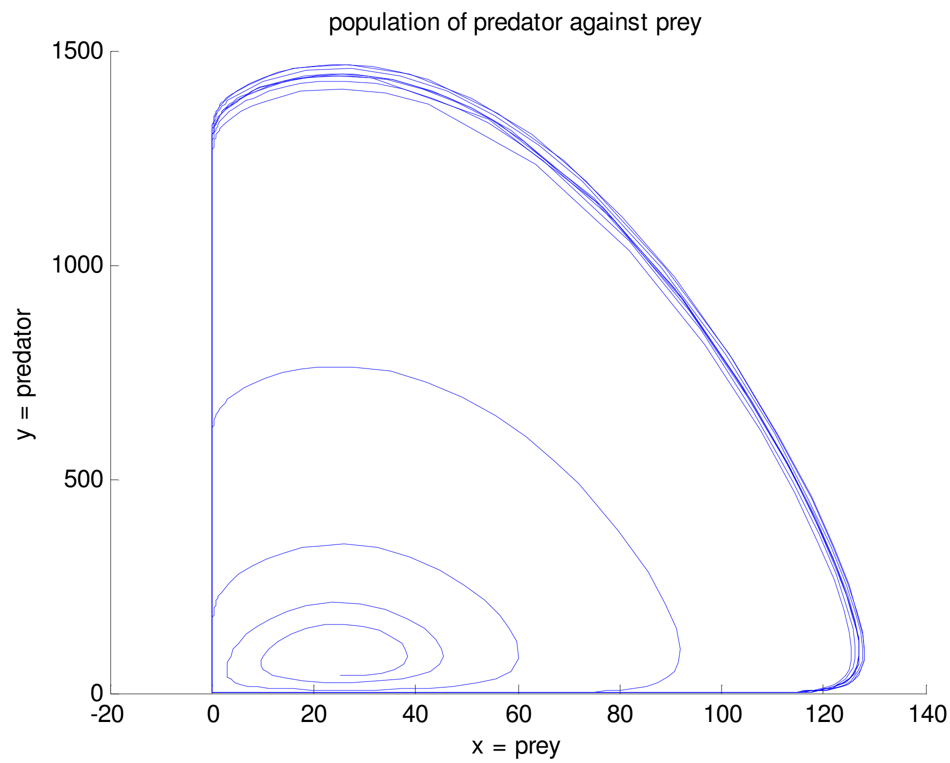
- when initial condition is [25,50]

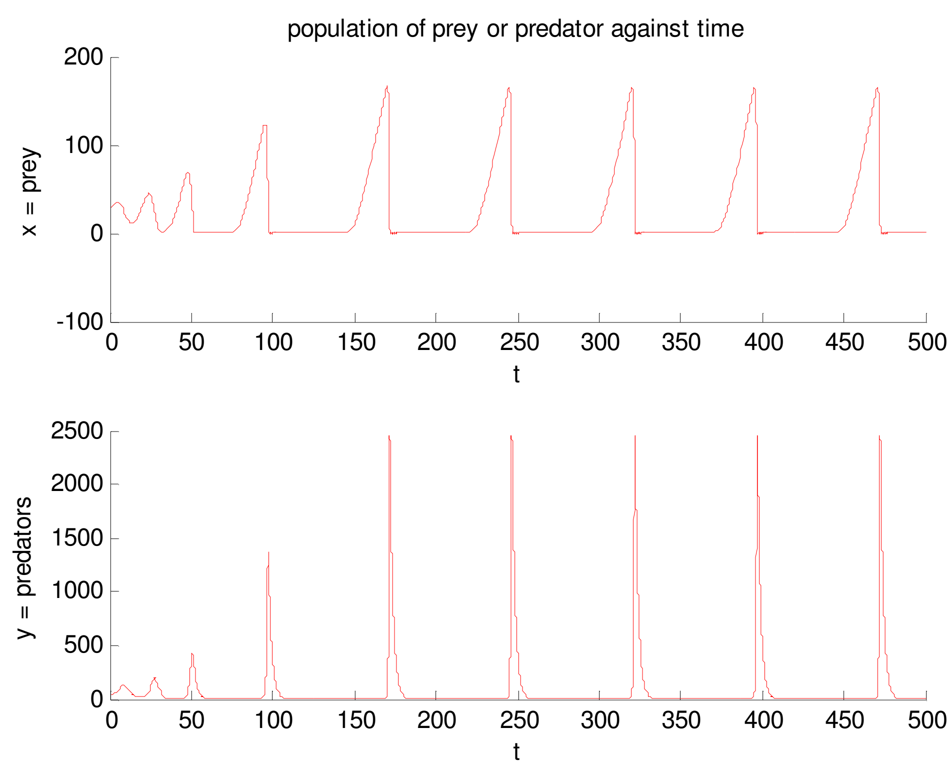




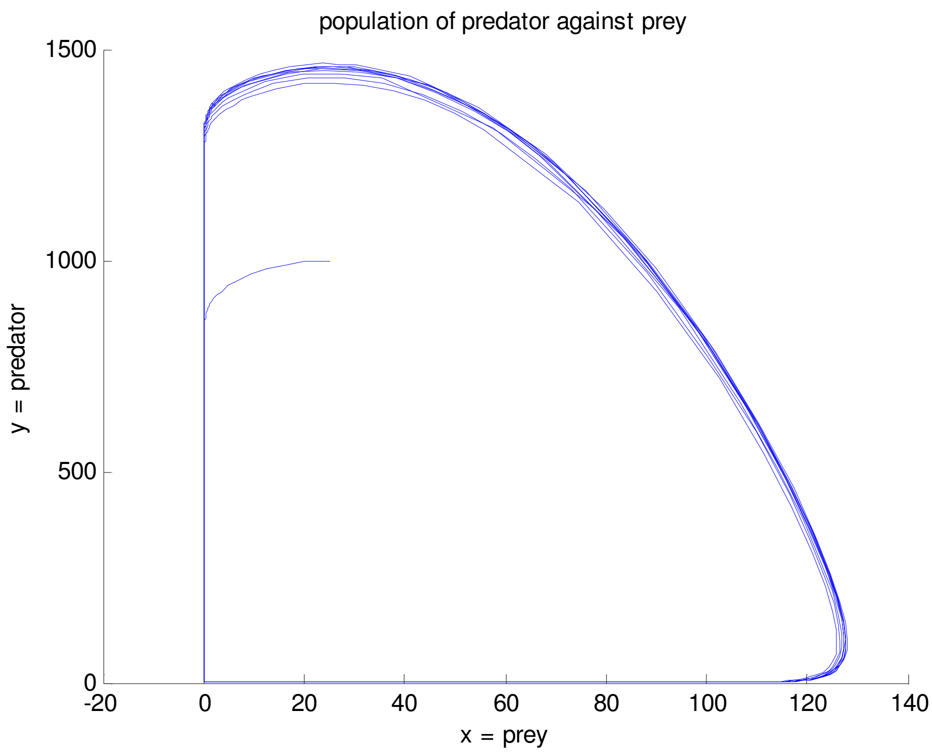
- when initial condition is [25,50]

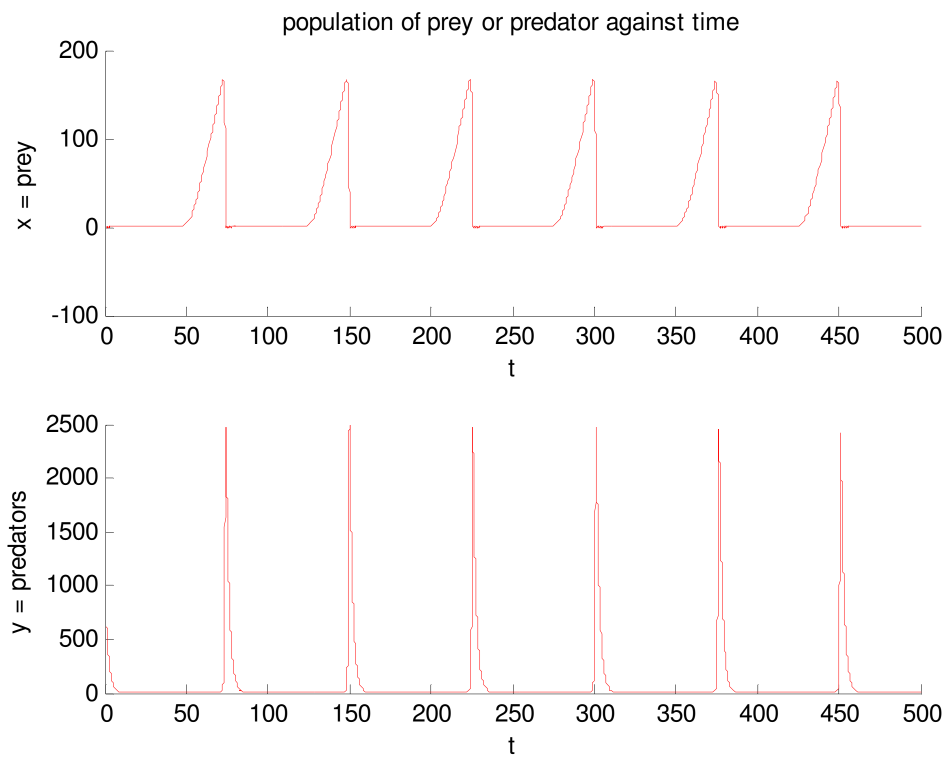
- a = 1 b = 0.1 c = 0.02 d = 0.5 V = 10 k = 15 m = 4 (k>m, unstable)





- when initial condition is [25,40]

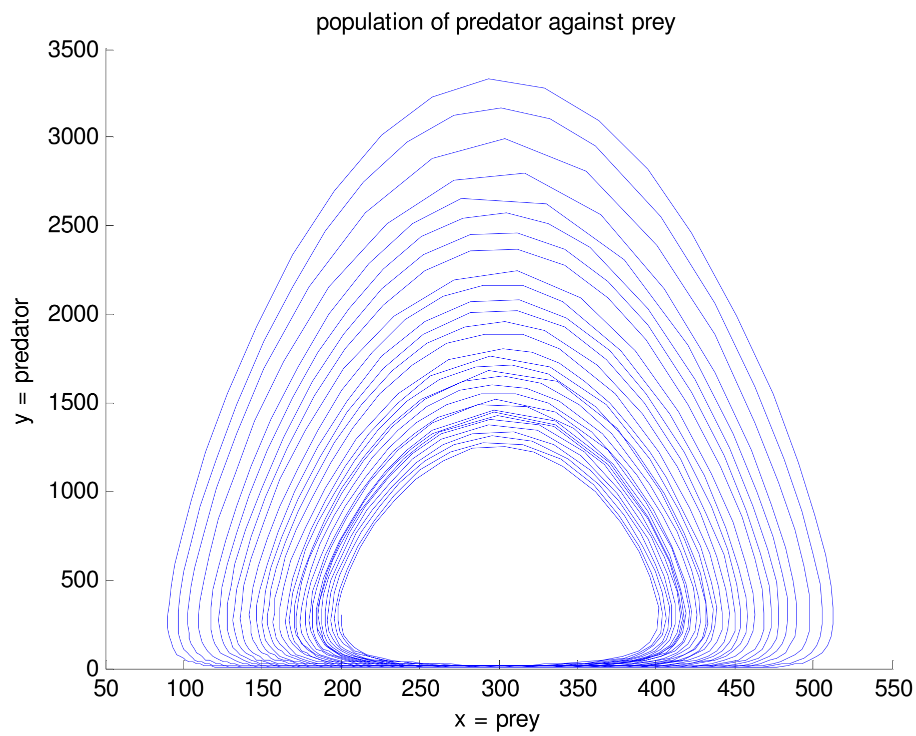


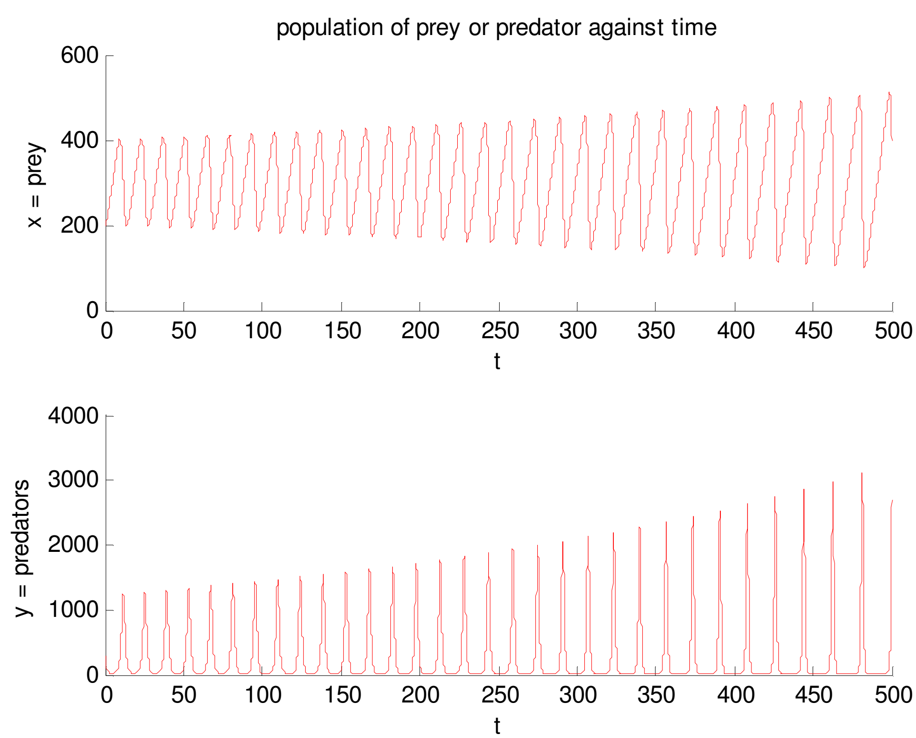


- when initial condition is [25,1000]

Now we need to put our stationary far apart to check the limit cycle, hence we set (d/c, V*(m*c+d)/b/(k*c+d))=(300,300)

thus set d=3, c=0.01, V=30, m=10, b=0.1, k=20 (k>m, unstable)

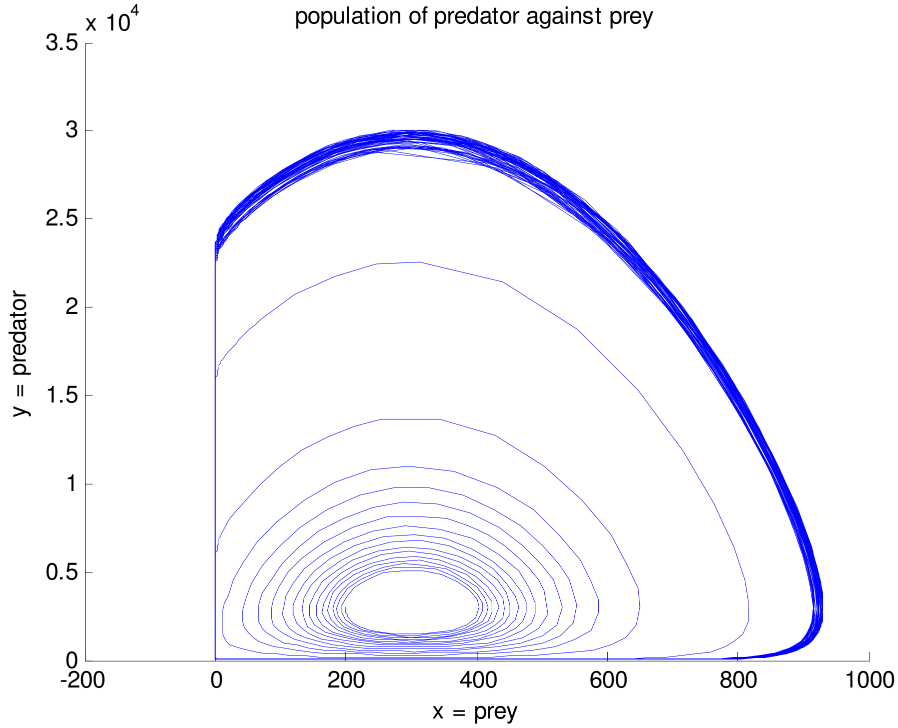


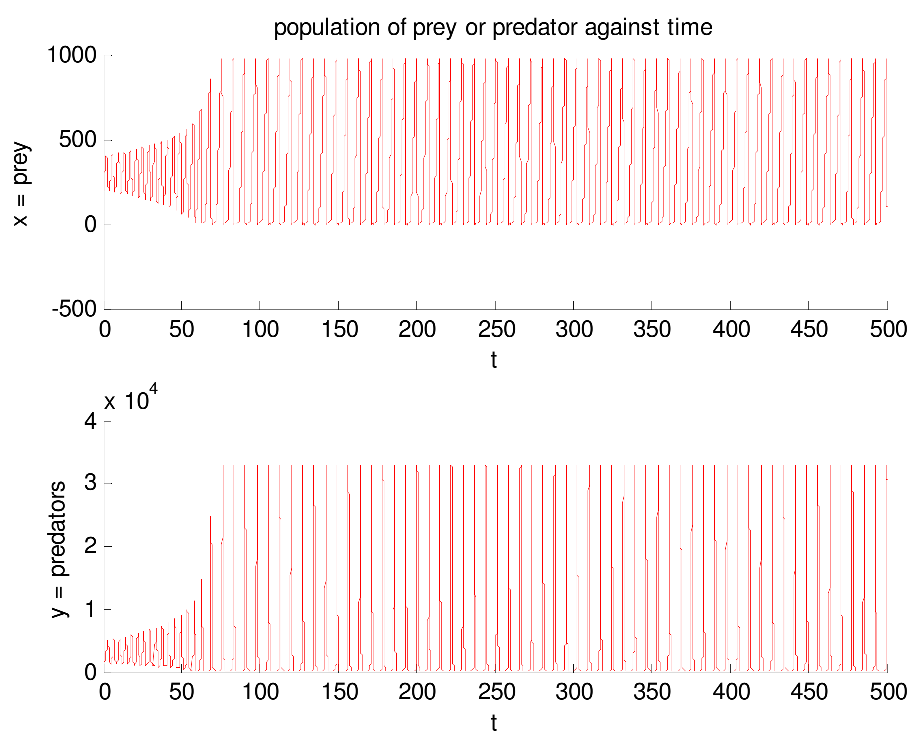


with the initial condition [200,300]

(d/c, V*(m*c+d)/b/(k*c+d))=(300,300)

thus set d=3, c=0.01, V=300, m=10, b=0.1, k=20 (k>m, unstable)





with the initial condition [200,3000]

In conclusion, it is difficult to put limit cycle away from axis by changing the stationary point only